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MIXED SPATIAL PROBLEMS OF ELASTICITY THEORY WITH A CIRCULAR LINE SEPARATING THE BOUNDARY CONDITIONS*

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Mixed problems for the Laplace equation in a half-space that occur in the theory of contact interaction and the theory of cracks are considered. The lines separating the boundary condition are considered to be circular, but the problem can be non-axisymmetric. Special integral relations are set up between the Fourier transform components of a harmonic function and its derivatives in the problems mentioned. The solution of a problem of an annular separation crack in an unbounded medium under non-axisymmetric loads is constructed as an example. Other examples are contained in /1-7/ and in the preprint** where the contact problem is considered.

The method used in this paper to construct the fundamental relationships is closest to that proposed in /3/. Different approaches to the construction of the solution of the mixed problems for a harmonic function in a half-space with circular interfacial lines are considered in a non-axisymmetric formulation in /1-4/.

1. Fundamental representations for a harmonic function. Let $f(r, \beta, x_3)$ be a harmonic function in the half-space $x_3 > 0$ ((r, β, x_3) are cylindrical coordinates). It can be represented in the form of the series

$$f = \sum_{n=-\infty}^{\infty} f_n \exp(in\beta) \tag{1.1}$$

$$f_n = \int_0^\infty A_n(q) J_n(qr) \exp(-qx_3) q\,dq$$

$$A_n(q) = \int_0^\infty f_n(r, 0) J_n(q, r) r\,dr$$

Let us first examine the case $n \geqslant 0$. We use well-known representations to transform expressions (1.1)

$$J_\nu \Gamma\left(\nu + \frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)\left(\frac{z}{2}\right)^{-\nu} = \int_{-1}^{1} (1-t^2)^{\nu-1/2} \cos zt\,dt, \quad \operatorname{Re}\nu > -\frac{1}{2} \tag{1.2}$$

$$J_\nu \Gamma\left(\frac{1}{2} - \nu\right)\left(\frac{z}{2}\right)^{\nu} = 2\int_1^\infty (t^2-1)^{-\nu-1/2} \sin zt\,dt, \quad 0 < \operatorname{Re}\nu < \frac{1}{2} \tag{1.3}$$

These representations can be considered as a Fourier transformation of functions $(1-t^2)^{\nu-1/2}$, $(t^2-1)^{-\nu-1/2}$ symmetric and antisymmetric with respect to the point $t = 0$.

We continue the left sides of (1.2) and (1.3) analytically, respectively, in the domain $\operatorname{Re}\nu < -1/2$, $\operatorname{Re}\nu > 1/2$. We will here consider right-hand sides as Fourier cosine and sine transforms of the generalized functions $(1-t^2)_+^{\nu-1/2}$, $(t^2-1)_+^{-\nu-1/2}$.

We set $\nu = -n$ in (1.2) and $\nu = n$ in (1.3) and take $z = rq$. Then

$$\frac{J_n \pi r^{-n}}{2(2n-1)!!} q^n = \int_0^\infty (r^2 - x^2)_+^{-n-1/2} \cos qx\,dx$$

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$$\frac{J_n \pi r^{-n} (-1)^n}{2(2n-1)!!} q^n = \int_0^\infty (x^2 - r^2)_+^{-n-1/2} \sin qx \, dx$$

Substituting these expressions into (1.1) and changing the order of integration, we obtain

$$f_n = r^n (2n-1)!! \int_0^\infty (r^2 - x^2)_+^{-n-1/2} \psi_n^{(1)}(x, x_3) \, dx \tag{1.4}$$

$$\psi_n^{(1)}(x, x_3) = \frac{2}{\pi} \int_0^\infty A_n(q) \exp(-qx_3) q^{-n+1} \cos qx \, dq \tag{1.5}$$

$$f_n = r^n (2n-1)!! (-1)^n \int_0^\infty (x^2 - r^2)_+^{-n-1/2} \psi_n^{(2)}(x, x_3) \, dx \tag{1.6}$$

$$\psi_n^{(2)}(x, x_3) = \frac{2}{\pi} \int_0^\infty A_n(q) \exp(-qx_3) q^{-n+1} \sin qx \, dq \tag{1.7}$$

From (1.1) for $A_n(q)$ we have $J_n(qr) \sim (qr)^n$ as $q \to 0$. Therefore, the function in the integrand of (1.5) and (1.7) is integrable and damps out as $x_3 \to \infty$. We shall still assume that $n \geqslant 0$. Then f_n in (1.4) and (1.6) damps out as $r \to \infty$, $x_3 \to \infty$. The functions $\psi_n^{(1)}(x, x_3)$, $\psi_n^{(2)}(x, x_3)$ here satisfy the two-dimensional Laplace equation and are analytic.

We now consider the following expressions for $f_n(r, x_3)$ and $\partial f_n(r, x_3)/\partial x_3$, that are necessary to construct the integral relations. We take the derivative with respect to x_3 in (1.4).

Then taking account of the integral $\psi_n^{(1)}$ within the limits from 0 to x we obtain by using the relationship (1.7)

$$\frac{\partial f_n}{\partial x_3}(r, x_3) = r^n (2n+1)!! \int_0^\infty (r^2 - x^2)_+^{-n-3/2} \psi_n^{(2)}(x, x_3) \, x \, dx \tag{1.8}$$

We will use the relations

$$\left(\frac{1}{x}\frac{\partial}{\partial x}\right)^n (x^2 - r^2)_+^{-1/2} = (-1)^n (2n-1)!! (x^2 - r^2)_+^{-n-1/2} \tag{1.9}$$

$$\left(\frac{1}{x}\frac{\partial}{\partial x}\right)^n \left[\frac{x^{2n+1}}{r^{2n}} (r^2 - x^2)_+^{-3/2}\right] = (2n+1)!! (r^2 - x^2)_+^{-n-3/2} x$$

to set up a connection between (1.8) and (1.6).

The first relation in (1.9) is obvious, the second is proved by mathematical induction.

We introduce the function

$$\Phi^{(2)}(x, x_3) = x \left(\frac{1}{x}\frac{\partial}{\partial x}\right)^n \left[\frac{\psi^{(2)}(x, x_3)}{x}\right]$$

Then by using (1.9), we obtain that (1.8) and (1.6) take the following form after integration by parts /3/

$$\frac{\partial f_n}{\partial x_3} = r^{-n} \int_0^\infty (r^2 - x^2)_+^{-3/2} x^{2n+1} \Phi_n^{(2)}(x, x_3) \, dx \tag{1.10}$$

$$f_n(r, x_3) = r^n \int_0^\infty (x^2 - r^2)_+^{-1/2} \Phi_n(x, x_3) \, dx \tag{1.11}$$

We will now use the relationships /3/

$$\frac{2}{\pi} \int_0^\infty (x_1^2 - r^2)_+^{-1/2} (r^2 - x^2)_+^{-1/2} r \, dr = H(x_1 - x) \tag{1.12}$$

Taking the integral by parts in (1.10), and then we multiply the relationship obtained by $r^n (x_1^2 - r^2)_+^{-1/2} r dr$, and (1.11) by $r^{-n} (r^2 - x_1^2)_+^{-1/2} r dr$ with (1.12) taken into account, we obtain two representations for the functions $\Phi_n^{(2)}(x, x_3)$, after integration between 0 and ∞ and differentiation of the second relationship with respect to x_1, and by comparing them we obtain the first integral relation

$$x_1^{2n+1}\int_{x_1}^{\infty}\frac{\partial}{\partial r}(r^{-n}f_n)W(r,x_1)\,dr=\int_0^{x_1}\frac{\partial f_n}{\partial x_3}W(x_1,r)\,r^{n+1}\,dr \tag{1.13}$$

The following notation has been introduced here: $W(r,\ x_1)\equiv(r^2-x_1^2)^{-1/2}$ and $W(x_1,r)\equiv(x_1^2-r^2)^{-1/2}$, which will henceforth be used.

To obtain the second integral relation we take the derivative with respect to x_3 in (1.6) and taking account of (1.5) and (1.7) we obtain

$$f_n=r^n(2n-1)!!\int_0^{\infty}(r^2-x^2)_+^{-n-1/2}\psi_n^{(1)}(x,x_3)\,dx \tag{1.14}$$

$$\frac{\partial f_n}{\partial x_3}=r^n(2n-1)!!\int_0^{\infty}(x^2-r^2)_+^{-n-1/2}\frac{\partial\psi_n^{(1)}}{\partial x}\,dx \tag{1.15}$$

We will use the relationship (1.9) and the relationship

$$\frac{\partial}{\partial x}\left(\frac{1}{x}\frac{\partial}{\partial x}\right)^{n-1}[x^{2n-1}(r^2-x^2)_+^{-1/2}]=(2n-1)!!\,(r^2-x^2)_+^{-n-1/2}r^{2n}$$

to set up the connection between (1.14) and (1.15).

The proof of these relationships is analogous to the proof of the relationships (1.9). Then, taking the integrals by parts in (1.14) and (1.15), and taking account of the last relationships, by introducing the function

$$\Phi_n^{(1)}(x,x_3)=\partial\,[(x^{-1}\partial/\partial x)^n\psi_n^{(1)}(x,x_3)]/\partial x$$

we obtain

$$f_n=r^{-n}\int_0^{\infty}(r^2-x^2)_+^{-1/2}x^{2n-1}\Phi_n^{(1)}\,dx \tag{1.16}$$

$$\frac{\partial f_n}{\partial x_3}=r^n\int_0^{\infty}(x^2-r^2)_+^{-1/2}\frac{\partial(\Phi_n^{(1)}x^{-1})}{\partial x}\,dx \tag{1.17}$$

Multiplying (1.16) and (1.17) by $r^n(x_1^2-r^2)_+^{-1/2}r\,dr$ and $r^{-n}(r^2-x_1^2)_+^{1/2}r\,dr$ respectively, and taking (1.12) into account we integrate between 0 and ∞. Then we differentiate the first of the relations obtained with respect to x_1, after which we arrive at two representations of the functions $\Phi_n^{(1)}(x_1,x_3)$, comparing which we obtain the second integral relation

$$D_n+x_1\int_0^{x_1}\frac{\partial}{\partial r}(f_nr^n)W(x_1,r)\,dr=-x_1^{2n}\int_{x_1}^{\infty}\frac{\partial f_n}{\partial x_3}r^{-n+1}W(r,x_1)\,dr$$

$$D_n=\lim_{r\to 0}(f_nr^n)$$

This last integral relation and the relationship (1.13) were obtained for the case $n\geqslant 0$. These relationships also hold for the case when $n<0$ by replacing n therein by $|n|$ /3/ and will have the form

$$x_1^{2m+1}\int_{x_1}^{\infty}\frac{\partial}{\partial r}(r^{-m}f_n)W(r,x_1)\,dr=\int_0^{x_1}\frac{\partial f_n}{\partial x_3}r^{m+1}W(x_1,r)\,dr \tag{1.18}$$

$$D_n+x_1\int_0^{x_1}\frac{\partial}{\partial r}(r^mf_n)W(x_1,r)\,dr=-x_1^{2m}\int_{x_1}^{\infty}\frac{\partial f_n}{\partial x_3}r^{1-m}W(r,x_1)\,dr \tag{1.19}$$

$$D_n=\lim_{r\to 0}(f_nr^m),\ m=|n|$$

Therefore, the relationships obtained connect the value of f_n and $\partial f_n/\partial x_3$. In particular, in the construction of the solution of the mixed problem of elasticity theory in the half-space $x_3 \geqslant 0$, when for $r > R$ we know that $\partial f_n/\partial x_3$, $x_3 = 0$, while for $r < R$, $f_n(r, 0)$, so it is convenient to use (1.19). If f_n is known for $x_3 = 0$ and $r > R$, while for $r < R$, $x_3 = 0$, $\partial f_n/\partial x_3$, then (1.18).

We now examine the more general case of the mixed problem with two interfacial lines of the boundary conditions. Let a boundary condition be given for $x_3 = 0$

$$f = \varphi_1(r, \beta),\ 0 \leqslant r \leqslant \beta,\ f = \varphi_2(r, \beta),\ r \geqslant a \qquad (1.20)$$
$$\frac{\partial f}{\partial x_3} = \chi(r, \beta), \quad R \leqslant r \leqslant a$$

We introduce the notation $\partial f/\partial x_3 = \chi_1(r, \beta)$, $0 \leqslant r \leqslant \beta$; $\partial f/\partial x_3 = \chi_2(r, \beta)$, $r \geqslant a$. Then we have a system of equations for $x_3 = 0$ from (1.18) and (1.19)

$$\int_y^R \chi_{1n} r^{1-m} W(r, y)\, dr + \int_a^\infty \chi_{2n} r^{1-m} W(r, y)\, dr = F_1(y) \quad 0 \leqslant y \leqslant R \qquad (1.21)$$
$$\int_0^R \chi_{1n} r^{1+m} W(y, r)\, dr + \int_a^y \chi_{2n} r^{1+m} W(y, r)\, dr = F_2(y) \quad y \geqslant a$$
$$F_1(y) = -\int_R^a \chi_n r^{1-m} W(r, y)\, dr - y^{-2m}\left[D_n + y\int_0^y \frac{\partial}{\partial r}(\varphi_{1n} r^m)\, W(y, r)\, dr\right]$$
$$F_2(y) = -\int_R^a \chi_n r^{m+1} W(y, r)\, dr + y^{2m+1}\int_y^\infty \frac{\partial}{\partial r}(\varphi_{2n} r^{-m})\, W(r, y)\, dr$$

To obtain the solution of system (1.21), we regularize it. We multiply the first equation by $(y^2 - x^2)^{-1/2} y\,dy$ and integrate it between x and R and then differentiate with respect to x. We multiply the second equation by $y\,dy\,(x^2 - y^2)^{-1/2}$ and integrate between a and x and then take the derivative with respect to x. We consequently obtain

$$\frac{\pi}{2}\chi_{1n} + x^m W(R, x)\int_a^\infty \frac{\chi_{2n} W^{-1}(r, R)\, r^{-m+1}}{r^2 - x^2}\, dr = \psi_1(x) \quad 0 \leqslant x \leqslant R \qquad (1.22)$$
$$\frac{\pi}{2}\chi_{2n} + x^{-m} W(x, a)\int_0^R \frac{\chi_{1n} W^{-1}(a, r)\, r^{m+1}}{x^2 - r^2}\, dr = \psi_2(x) \quad x \geqslant a$$
$$\psi_1(x) = x^m \frac{d}{dx}\int_x^R F_1 y W(y, x)\, dy = x^m\left(F_1(R)\, W(R, x) - \int_x^R F_1' W(y, x)\, dy\right)$$
$$\psi_2(x) = x^{-m}\frac{d}{dx}\int_a^x F_2 y W(x, y)\, dy = x^{-m}\left(F_2(a)\, W(x, a) + \int_a^x F_2' W(x, y)\, dy\right)$$

We note that in the special case when $\varphi_1(r, \beta) \equiv 0$, $0 \leqslant r \leqslant R$, $\varphi_2(r, \beta) \equiv 0$, $r \geqslant a$ the expression for the function $\psi_1(x)$, $\psi_2(x)$ in (1.22) simplifies to

$$\psi_1(x) = x^m W(R, x)\int_R^a \frac{\chi_n r^{-m} W^{-1}(r, R)}{r^2 - x^2}\, r\, dr \quad 0 \leqslant x \leqslant R$$
$$\psi_2(x) = x^{-m} W(x, a)\int_R^a \frac{\chi_n W^{-1}(a, r)\, r^{m+1}}{x^2 - r^2}\, dr \quad x \geqslant a$$

The system (1.22) already does not contain the singularity in the integrand for $r = x$. We will now examine the question of the method of solving this system. To do this, we expand $(r^2 - x^2)^{-1}$ and $(x^2 - r^2)^{-1}$, respectively in the integrands of (1.22) in powers of $r^{-2}x^2$ and r^2x^{-2} and substitute into (1.22). We hence obtain

$$\chi_{1n} = \frac{2}{\pi}\left[\psi_1(x) - \sum_{k=0}^{\infty} A_k x^{2k+m} W(R, x)\right], \quad 0 \leqslant x \leqslant R \qquad (1.23)$$
$$\chi_{2n} = \frac{2}{\pi}\left[\psi_2(x) - \sum_{k=0}^{\infty} C_k x^{-2(k+1)-m} W(x, a)\right], \quad x \geqslant a$$

$$A_k = \int_a^\infty \chi_{2n} (W^{-1}(r, R)\, r^{-2k-1+m}\, dr$$

$$C_k = \int_0^R \chi_{1n} W^{-1}(a, r)\, r^{2k+m+1}\, dr$$

We multiply the first equation of system (1.23) by $x^{2l+m}(a^2 - x^2)^{1/2} x dx$, and the second by $x^{-2(l+1)-m}(x^2 - R^2)^{1/2} x dx$ and integrate, respectively, between 0 and R and between a and ∞. We obtain the system of algebraic equations

$$\frac{\pi}{2} L_l \left(\frac{a}{R}\right)^{2l+m+3} + \sum_{k=0}^{\infty} M_k F_{k+l} = \psi_{1,l}^{(0)} \tag{1.24}$$

$$\frac{\pi}{2} M_l \left(\frac{a}{R}\right)^{2l+m} + \sum_{k=0}^{\infty} L_k F_{k+l} = \psi_{2,l}^{(0)}$$

$$F_{k+l} = \int_0^1 (\varepsilon^2 + \eta^2)^{1/2} (1 - \eta^2)^{k+l+m}\, d\eta, \quad \varepsilon = (a^2/R^2 - 1)^{1/2}$$

$$F_{k+l}^{(1)} = F_{k+l} R^{2(k+l+m+1)}, \quad A_l = M_l R^{-(2l+m-1)}$$

$$F_{k+l}^{(2)} = F_{k+l} a^{-2(k+l+m)-3} R, \quad C_l = L_l a^{2l+m+3}$$

$$\psi_{1,l}^{(0)} = \int_0^1 \Phi_1^{(0)}(\xi_1)(1 - \eta^2)^{l+m/2} (\varepsilon^2 + \eta^2)^{1/2}\, d\eta$$

$$\psi_{2,l}^{(0)} = \int_0^1 \Phi_2^{(0)}(\xi_2)(1 - \eta^2)^{l+m/2-1} (\varepsilon^2 + \eta^2)^{1/2}\, d\eta$$

$$\Phi_1(x) = R\Phi_1^{(0)}(\xi_1), \quad \Phi_2(x) = a\Phi_2^{(0)}(\xi_2)$$

$$\psi_1(x) = \Phi_1(x)(R^2 - x^2)^{-1/2}, \ \psi_2(x) = \Phi_2(x)(x^2 - a^2)^{-1/2}$$

$$\xi_1 = (1 - \eta^2)^{1/2} R, \ \xi_2 = a(1 - \eta^2)^{-1/2}$$

The theory of infinite systems is discussed in /9/, for example. The solution of the infinite system of algebraic Eqs.(1.2) obtained will be examined below by the method of reduction in the example of mixed problems of elasticity theory /9/.

2. ***Fundamental representations of elasticity theory for a normal problem.*** Examples of the computation. Let a disc or annular crack Ω be in the $x_3 = 0$ plane in an $X_1X_2X_3$ system of coordinates for normal loads given on its surfaces. The normal displacement $u_3 = u$ and stress σ_{33} in the $x_3 = 0$ plane will be described in terms of the harmonic function f

$$\sigma_{33} = 2\mu\left(1 - x_3 \frac{\partial}{\partial x_3}\right)\frac{\partial f}{\partial x_3} \tag{2.1}$$

$$u = \left(2 - 2\nu - x_3 \frac{\partial}{\partial x_3}\right) f$$

The expression for the remaining displacement and stress components is in /3/.

The boundary-value problem for the half-space $x_3 < 0$ has the form

$$\partial f/\partial x_3 = \sigma(r, \beta), \ (x_1, x_2) \in \Omega \tag{2.2}$$

$$f = u(r, \beta)/[2(1 - \nu)]$$

As an illustration we consider the case of a disc crack of radius a. We use the relationships (1.18) and taking account of (2.2) obtain

$$\int_x^a \frac{\partial}{\partial y}(y^{-|n|} u_n) W(y, x)\, dy = \frac{(1-\nu)}{\mu} x^{2|n|-1} \int_0^x \sigma y^{|n|+1} W(x, y)\, dy, \quad 0 \leqslant x \leqslant a \tag{2.3}$$

Multiplying (2.3) by $x dx (x^2 - y^2)^{1/2}$ and integrating between r and a we obtain

$$u_n(r, 0) = -\frac{2(1-\nu)}{\pi\mu} r^{|n|} \int_r^a x^{-2|n|} W(x, r)\, dx \int_0^x \sigma_n y^{|n|+1} W(x, y)\, dy \tag{2.4}$$

$$0 \leqslant r \leqslant a$$

This result agrees with that obtained in /3/.

We will now examine the case of an exterior crack of radius a, $\Omega = (a, \infty)$. We use the relationship (1.19) and taking account of (2.2), we have

$$\int_a^x \frac{\partial}{\partial y} (y^{|n|} u_n) W(x, y)\, dy = \frac{(1-\nu)}{\mu} x^{2|n|-1} \int_x^\infty \sigma_n y^{-|n|+1} W(y, x)\, dy$$

$$x \geqslant a$$

Multiplying this relationship by $x dx (r^2 - x^2)^{-1/2}$ and integrating between a and r we obtain

$$u_n = -\frac{2(1-\nu)}{\pi\mu} r^{-|n|} \int_a^r x^{2|n|} W(r, x)\, dx \int_x^\infty \sigma_n y^{-|n|+1} W(y, x)\, dy \quad r \geqslant a \tag{2.5}$$

Expressions (2.4) and (2.5) determine the Fourier components of the expansion of the displacement in terms of the Fourier components of the stresses. Multiplying these expressions by $e^{in\varphi}$ and summing while taking into account

$$u = \sum_{n=-\infty}^{\infty} u_n e^{in\varphi}$$

$$\sigma_n = \frac{1}{2\pi} \int_0^{2\pi} \sigma(r, \beta)\, e^{-in\beta}\, d\beta$$

we obtain

$$u(r, \varphi) = -\frac{(1-\nu)}{\pi^2\mu} \int_0^{2\pi} \int_r^a \int_0^x \sigma(y, \beta)\, ZW(x, r)\, W(x, y)\, y\, dy\, dx\, d\beta \tag{2.6}$$

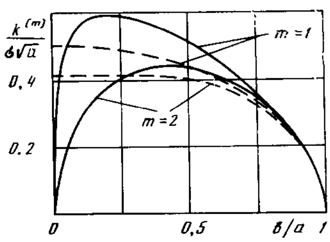
$$u(r, \varphi) = \frac{(1-\nu)}{\pi^2\mu} \int_0^{2\pi} \int_a^r \int_x^\infty \sigma(y, \beta)\, ZW(r, x)\, W(y, x)\, y\, dy\, dx\, d\beta$$

$$Z = (x^4 - y^2 r^2)/(r^2 y^2 - 2ryx^2 \cos(\beta - \varphi) + x^4)$$

The first relationship in (2.6) is identical with that obtained in /3/.

We now examine the case of a ring crack $b \leqslant r \leqslant a$ and a load $\sigma(r, \beta, 0) = \sigma_0(r) + \sigma_m e^{im\beta}$

In this case the displacement will have the form



$$u(r, \beta) = u_0(r) + u_m e^{im\beta}$$

Correspondingly, the intensity factors are $K = K^{(0)} + K^{(m)} e^{im\beta}$.

The dependence of K_m on b/a for $\sigma_m = \sigma_0 = \sigma = \text{const}$ and $m = 1, 2$ is displayed in the figure for the crack boundary for $r = b$ (the solid lines) and $r = a$ (the dashed lines).

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